Topological Properties of One-Dimensional Non-Pisot Substitutions

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Abstract

We generalize the notion of phason introduced for quasiperiodic structures created using the Cut&Project method. We show that the phason is related to the number of rearrangements of rows in a $2d$ quasiperiodic system. Under this rearrangements some properties of the diffraction pattern remain unchanged. When the length of the system is a Fibonacci number, periodic boundary conditions can be used, so that the Bragg peaks are related to the Chern numbers. We generalize this result to a general two-letter substitution and give the rules for a characteristic length of a non Pisot substitution. We then show that just like in the case of Fibonacci substitutions this length relates to periodic boundary conditions. Finally, we show that the same set of Chern numbers can be obtained using a set of excange rules related by gauge transformations and valid ever for Pisot or non-Pisot substitutions.