Nonlinearity-induced frequency fluctuations in a nanomechanical resonator

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Fluctuations in nanomechanical resonators are of primary interest since they address both applied¹ and fundamental issues. On the fundamental side, a nanomechanical resonator can be studied as a model mechanical (an-)harmonic oscillator dissipatively coupled to a thermal bath. The latter ensures through fluctuation-dissipation theorem that the resonator experiences a stochastic Langevin force, i.e. thermal fluctuations in position. Meanwhile, geometric nonlinearities enable to dispersively couple two distinct modes of the same resonator while allowing a good



SiN nanostring 300 µm long, 300 nm wide, 190 nm thick (gate not used)

implementation of the Duffing model for a single mode. One can then study complex dynamics arising both from thermal fluctuations and nonlinearities in the same mechanical system³. Indeed, the position fluctuations of a given mechanical mode are converted in frequency fluctuations through the nonlinearities, both for the mode itself and the other modes. The magnitude of these fluctuations depends then on the thermal energy stored in the mode, even though no additional energy flow is created⁴.

We experimentally investigated the nonlinear interaction between a driven nanomechanical mode and another non-driven mode brought to high effective temperatures, by measuring deviations to the standard mechanical susceptibility of the driven mode⁴. Different regimes arise, depending on the thermal energy of the hot mode, i.e., the magnitude of the frequency noise. We extended the study to a single mechanical mode, where the driven mode is coupled to its own Brownian motion. We showed that, while similar at first sight, the two situations are qualitatively and quantitatively different. A theoretical model in good agreement with our data highlighted the slow, non-Gaussian nature of the frequency fluctuations. Building on this proof of principle, we derived, with minimal assumptions, a fundamental limit on the frequency stability⁵ of a multimode mechanical resonator set by its nonlinearities.

^{1.} Hanay M. S. et al., Single-protein nanomechanical mass spectrometry, Nature Nanotechnology 7, 602-608, 2012

^{2.} Gieseler J. et al., Thermal nonlinearities in a nanomechanical resonator, Nature Physics 9, 806-810, 2013

^{3.} Maillet O. et al., Classical decoherence in a nanomechanical resonator, accepted in New. J. Phys., 2016

^{4.} Maillet O. et al., in preparation

^{5.} Sansa M. et al., Frequency fluctuations in silicon nanoresonators, Nature Nanotechnology, 2016