

## Multiple scattering of ultrasound in bubbly liquids and negative refraction

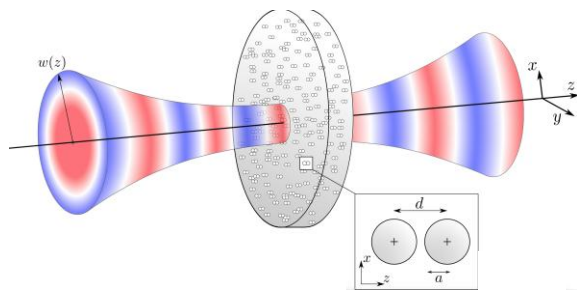
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A bubble in water exhibits a low-frequency monopolar resonance, the so-called Minnaert resonance<sup>1</sup>, which makes a distribution of bubbles an ideal system to study strong multiple scattering of acoustic waves. Following the seminal work of Foldy<sup>2</sup>, a lot of different approaches, such as the ones from Keller or Waterman and Truell, have been developed to infer the effective acoustic properties of a bubbly liquid. Here we confront the predictions of these different approaches to numerical results obtained with a multiple scattering theory (MST) code that fully incorporates the multiple scattering effects. Based on this study, we show the importance to take into account the interactions between neighboring bubbles to predict the transmission properties of a bubble slab.

As a possible application and following a recent idea suggested by Kaina et al.<sup>3</sup>, we show that negative refraction can be achieved with a slab filled with dimers of bubbles in water (cf. figure). It



Numerical study of acoustic wave propagation through a set of dimers of bubbles

is well-known that a set of identical bubbles in water exhibits a large gap above the Minnaert resonance. This gap, which is referred to as a hybridization gap, originates from the coupling of the wave propagating in water with the individual bubble resonance. In the gap frequency range, the effective compressibility is negative. When individual bubbles are replaced by dimers of bubbles, multiple scattering between the two bubbles in the dimer creates a dipolar resonance that overlaps the monopolar one. This dipolar resonance is responsible for the opening of a propagating band in

the hybridization gap, the slope of which being negative. That is, such a bubble metamaterial presents a negative refraction index. Interestingly, we show that negative refraction resists a random positioning of the dimers.

1. M. Minnaert, *Philos. Mag. J. Sci.* 16, 235, 1933

2. L. Foldy, *Phys. Rev.* 67, 107, 1945

3. N. Kaina et al., *Nature* 525, 77, 2015